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CLASIFICACION INIS Y DESCRIPTORES

A14
PLASMA WAVES
SPHERICAL CONFIGURATION
HOT PLASMA
HOMOGENEOUS PLASMA
COLLISIONAL PLASMA
LASER PRODUCED PLASMA

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NOTA

El presente trabajo corresponde al Informe #4 del equipo ii) Propagación de Ondas de Choque en Medios Ultradensos, del Subprograma de la JEN sobre Confinamiento Inercial del Proyecto Laser-Fisión-Fusión, descrito en el Informe JEN 351.

Local deposition of energy in fluids with nonlinear heat conduction may give raise to thermal waves and negligible convection.[1] In particular, when energy per unit area and time $\phi = \phi_o t/\tau$ is deposited in a given plane within a plasma, where conductivity is $K=\overline{K}T^{5/2}$, $\overline{K} \simeq$ constant, a thermal wave develops if $\alpha \equiv (9k/4m_i)(k^2\tau n_o{}^2/\phi_o\overline{K})^{2/3}$ is small (m_i is the ion mass, n_o the electron density, and k Boltzmann's constant).[2,3] It should be noticed that if α is too small, local equilibrium will not hold and heat conduction will be anomalous.[3,4]

In laser-plasma applications, radiation absorption may be limited to a thin, spherical layer. Assuming a monotonously growing pulse

$$\phi = \phi_o\, g(t/\tau), \qquad g(1) = 1 \quad ,$$

a spherical wave should develop for a broad class of pulse shapes and a moderately small α. Considering for simplicity an uniform plasma and absorption at $r = R$, the electron energy equation reads

$$\frac{3}{2} n_o k \frac{\partial T}{\partial t} = \frac{\overline{K}}{r^2} \frac{\partial}{\partial r}\left(r^2 T^{5/2} \frac{\partial T}{\partial r}\right) + \phi_o\, g(t/\tau)\, \delta(r-R) \quad ; \qquad (1)$$

then, introducing dimensionless variables

$$\hat{T} = T/T_o, \quad \hat{t} = t/\tau \;, \quad \hat{x} = (r-R)/w\tau$$

and setting[3]

$$T_o = (2\phi_o{}^2\tau/3kn_o\overline{K})^{2/9} \;, \quad w\tau \equiv \varepsilon R = (2\tau/3kn_o)^{7/9}\overline{K}^{2/9}\phi_o{}^{5/9}$$

Eq. (1) becomes

$$\frac{\partial \hat{T}}{\partial \hat{t}} = \frac{1}{(1+\varepsilon\hat{x})^2} \frac{\partial}{\partial \hat{x}}\left[(1+\varepsilon\hat{x})^2 \hat{T}^{5/2} \frac{\partial \hat{T}}{\partial \hat{x}}\right] + g(\hat{t})\,\delta(\hat{x}) \;. \qquad (2)$$

The initial and boundary conditions are

$$\hat{T} = 0 \text{ at } \hat{t} = 0, \qquad \hat{T} = \hat{T}^{5/2}\partial\hat{T}/\partial\hat{x} = 0 \quad \text{at } \hat{x} = \hat{x}_f \quad ,$$

$\hat{x}_f$ being either the inner ($\hat{x}_{in}$) or outer ($\hat{x}_{out}$) wavefront; it may be shown that $\hat{T} \sim (1-\hat{x}/\hat{x}_f)^{2/5}$ near $\hat{x}_f$.[1] In addition, the last term in (2) may be dropped if use is made of condition

$$\hat{T}^{5/2}\partial\hat{T}/\partial\hat{x} \Big|_{0^-}^{0^+} = -g(\hat{t}) \ . \tag{3}$$

For short times, curvature effects will be small. Expanding in powers of $\hat{t}$ and setting $g = \hat{t}^p$ we find to second order (quasiplanar approximation)

$$\begin{aligned} \hat{x}_f &= \pm\, \hat{t}^{(7+5p)/9}\, \xi_1 + \hat{t}^{(14+10p)/9}\, \xi_2 \\ \hat{T} &= \hat{t}^{(2+4p)/9}\Theta_1(s) \pm \hat{t}^{1+p}\Theta_2(s) \ , \qquad s \equiv \hat{x}/\hat{x}_f \quad ; \end{aligned} \tag{4}$$

the upper (lower) sign should be used for $\hat{x} > 0$ and $\hat{x}_{out}$ ($\hat{x} < 0$ and $\hat{x}_{in}$). We notice that the planar (lowest order) solution is symmetric, while the second order correction is antisymmetric. An excellent approximation to ξ_1 and Θ_1 is

$$\xi_1 \doteq (4/5)[7/8(1+p)]^{7/9} \ , \qquad \Theta_1 = [7/8(1+p)]^{2/9}(1-s)^{2/5} . \tag{5}$$

Both ξ_2 and Θ_2 [which behaves as $s(1-s)^{2/5}$ roughly] are negative; thus, curvature effects speed up $\hat{x}_{in}$ and slow down $\hat{x}_{out}$, and make the inner wave more step-like.

A really useful solution is provided by an integral method.

Multiplying Eq. (2) by $(1+\varepsilon\hat{x})^q$ and integrating between $\hat{x}_{in}$ and $\hat{x}_{out}$ we get

$$\frac{d}{d\hat{t}}\int_{\hat{x}_{in}}^{\hat{x}_{out}}(1+\varepsilon\hat{x})^q\hat{T}d\hat{x} = \frac{2}{7}\;\varepsilon^2(q-1)(q-2)\int_{\hat{x}_{in}}^{\hat{x}_{out}}(1+\varepsilon\hat{x})^{q-2}\hat{T}^{7/2}d\hat{x}+g\;;$$

for q = 1 and q = 2, we arrive at

$$\int_{\hat{x}_{in}}^{\hat{x}_{out}}(1+\varepsilon\hat{x})\hat{T}d\hat{x} = \int_0^{\hat{t}} g(\hat{t}')d\hat{t}' \;, \tag{6}$$

$$\int_{\hat{x}_{in}}^{\hat{x}_{out}}(1+\varepsilon\hat{x})\hat{x}\,\hat{T}\,d\hat{x} = 0 \;. \tag{7}$$

We then try an asymmetric profile

$$\begin{aligned}\hat{T}(\hat{t},\hat{x}) &= \hat{T}_R(\hat{t})\;[1-\hat{x}/\hat{x}_{out}(\hat{t})]^{2/5} \;, \quad 0<\hat{x}<\hat{x}_{out}\\ &= \hat{T}_R(\hat{t})\;[1-\hat{x}/\hat{x}_{in}(\hat{t})]^{2/5} \;, \quad \hat{x}_{in}<\hat{x}<0\end{aligned} \tag{8}$$

which behaves properly near either wavefront, and goes over to the planar profile (5) when $\hat{t}\to 0(\hat{x}_f\to 0)$; the three unknown functions of $\hat{t}$ may be obtained by using Eqs. (6), (7) and (3). Defining $\sigma\equiv\hat{x}_{out}/(-\hat{x}_{in})$, we arrive at an implicit equation for $\sigma(\hat{t},\varepsilon)$

$$\varepsilon^{9/7}\,\frac{\int_0^{\hat{t}} g d\hat{t}'}{g^{2/7}} = G(\sigma)\equiv\left[\frac{17(1-\sigma)}{1-\sigma+\sigma^2}\right]^{9/7}\frac{(1+\sigma)^{5/7}\sigma^{2/7}}{7\times 2^{11/7}}\left[1-\frac{17}{24}\,\frac{(1-\sigma)^2}{1-\sigma+\sigma^2}\right] \;, \tag{9}$$

and then obtain $\hat{x}_{out}$, $\hat{x}_{in}$ and $\hat{T}_R$ parametrically

$$\hat{x}_{out} = -\sigma\hat{x}_{in} = \frac{17}{10}\,\varepsilon^{-1}\,\frac{\sigma(1-\sigma)}{1-\sigma+\sigma^2} \;, \tag{10}$$

$$\hat{T}_R = \left[\frac{17}{4}\,\frac{g}{\varepsilon}\,\frac{\sigma(1-\sigma)}{(1+\sigma)(1-\sigma+\sigma^2)}\right]^{2/7} . \tag{11}$$

Figure 1 shows $G(\sigma)$ for $\sigma^* < \sigma < 1$, $\sigma^* \simeq 0.557$ being the value of σ when the inner front arrives at the origin ($\varepsilon\hat{x}_{in} = -1$), which follows from (10).

There is a critical value of ε, ε_{cr}, for which $\sigma = \sigma^*$ when $\hat{t} = 1$. From (9), clearly,

$$\varepsilon_{cr} = [G(\sigma^*)/\int_o^1 g d\hat{t}]^{7/9} \quad ;$$

ε_{cr} depends only on the pulse shape. For $\varepsilon < \varepsilon_{cr}$, the pulse ends before the inner front arrives at the origin, the solution found above being valid for $\sigma_F < \sigma < 1$, where σ_F is the value of σ at $\hat{t} = 1$

$$G(\sigma_F) = (\varepsilon/\varepsilon_{cr})^{9/7}\, G(\sigma^*) \quad ;$$

the final values of $\hat{T}_R$, $\hat{x}_{out}$, and $\hat{x}_{in}$ as functions of ε follow from (10) and (11). If $\varepsilon << \varepsilon_{cr}$, the analysis could be carried out for most of the time after the pulse ends as if energy deposition were instantaneous. For $\varepsilon > \varepsilon_{cr}$, the inner front arrives at the origin at a time $\hat{t}_F < 1$, given by

$$[g(\hat{t}_F)]^{-2/7}\int_o^{\hat{t}_F} g d\hat{t} = (\varepsilon_{cr}/\varepsilon)^{9/7}\int_o^1 g d\hat{t} \quad ;$$

at that time $\hat{x}_{out} = \sigma^*/\varepsilon$ independently of pulse shape.

To check solution (9)-(11) we integrated Eq. (2) times $\hat{T}^{3/2}(1+\varepsilon\hat{x})^2$ from $\hat{x}_{in}$ to $\hat{x}_{out}$, comparing with unity the ratio of both sides; it appears that for $p = 0(1)$ the solution is accurate until $r_{in} = R/4$, say. In fact, comparison to the quasiplanar approximation suggests that solution (9)-(11) remains valid all the way till the wave arrives at the origin: For $g = \hat{t}^p$, and using (9), Eqs. (10) and (11) become

$$\hat{T}_R = [\frac{7}{8(1+p)}]^{2/9} \hat{t}^{(2+4p)/9} [\frac{4\sigma(1+\sigma)^{-2}}{1-17(1-\sigma)^2/24(1-\sigma+\sigma^2)}]^{2/9} \quad (12)$$

$$\hat{x}_{out} = -\sigma\hat{x}_{in} = \frac{4}{5}[\frac{7}{8(1+p)}]^{7/9}\hat{t}^{(7+5p)/9}[\frac{2^{5/7}\sigma(1+\sigma)^{-5/7}}{1-17(1-\sigma)^2/24(1-\sigma+\sigma^2)}]^{7/9} \quad (13)$$

Since $\Theta_2(0)=0$, Eqs. (4) and (5) yield $\hat{T}(\hat{x} = 0)$ in excellent agreement with (12) [the last factor, function of σ, in (12) lies between 0.977 and 1 for $\sigma^* < \sigma < 1$]. Agreement for $\hat{x}_{out}$ is also excellent; Fig. 2 shows $\hat{x}_{out}$ (for $p = 1$), according to Eq. (4) and to Eqs. (13) and (9) with $\varepsilon = 3.5$. Also shown is the planar approximation, and results for $\hat{x}_{in}$; reasonaby, disagreement is largest for $\hat{x}_{in}$. We would expect $\hat{x}_{in}$ near the end to become steeper than the solid line in Fig. 2; it is clear from the figure, however, that this cannot affect noticeably the time of arrival of $\hat{x}_{in}$ to the origin.

ACKNOWLEDGMENTS

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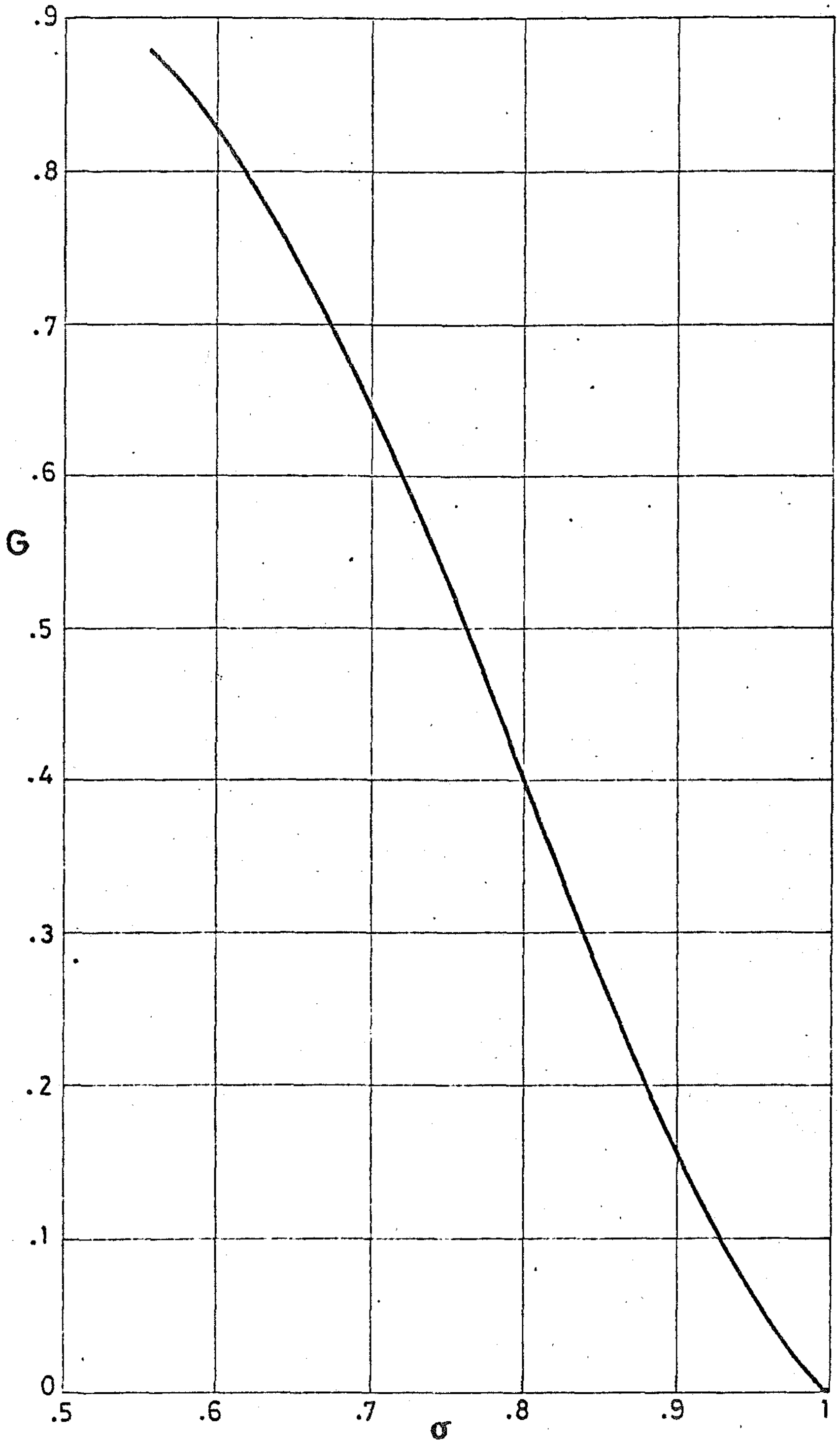
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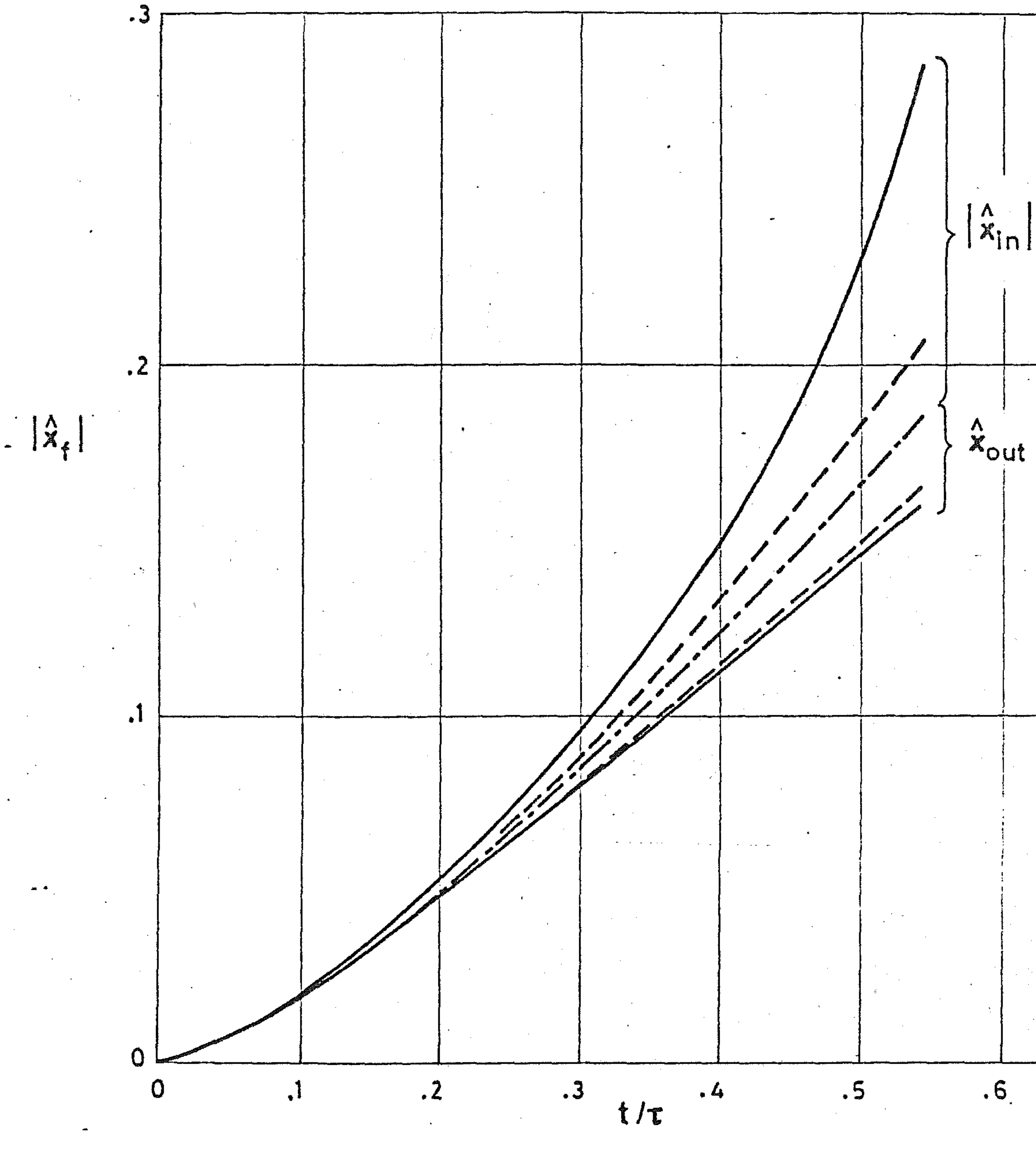

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"Onda térmica esférica en un plasma producida por un pulso de laser".

SANMARTIN, J.R.; MONTAÑES, J.L. (1978) 9 pp. 2 figs. 4 refs.

Se analiza, mediante un método integral, la onda térmica producida en un plasma uniforme por la absorción de un pulso de energía en una superficie esférica, de tal modo que la convección sea despreciable. Se comprueba la buena aproximación de los resultados

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